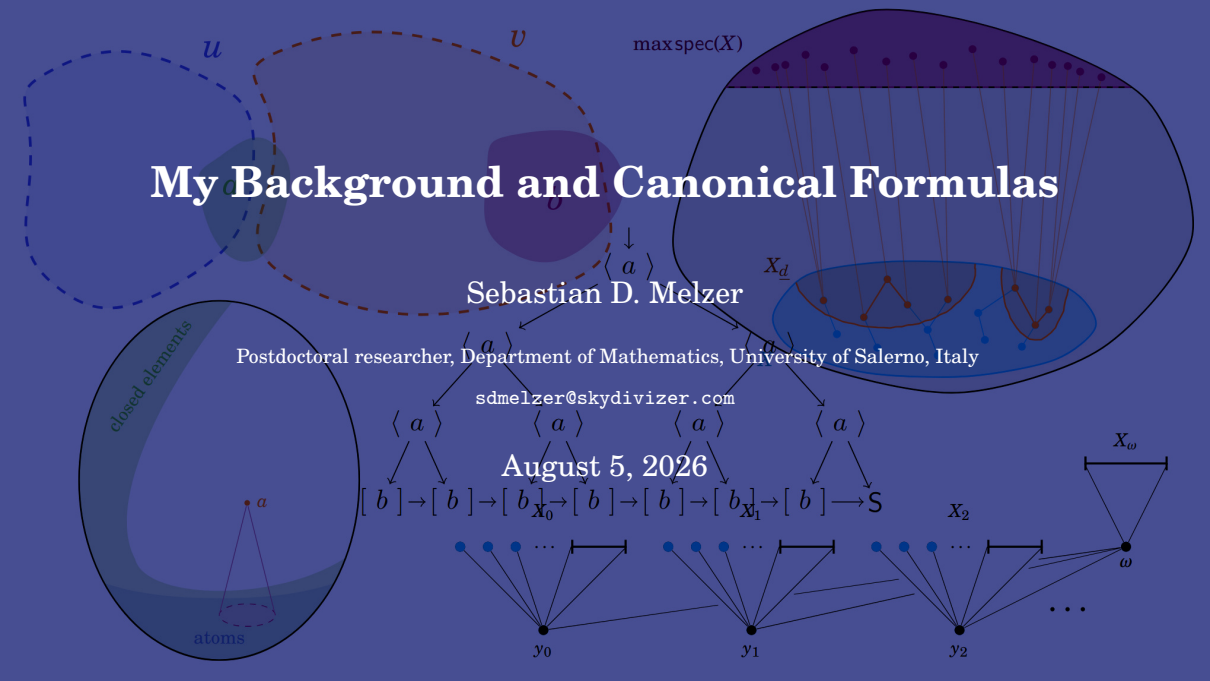


My Background and Canonical Formulas

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Some background

Let me begin at the very beginning.

I was born in Tübingen, a university town in Germany.

My parents studied there.

I remember almost nothing about it.



Tübingen from above

Some background

My parents later moved to the Netherlands, to a small town called Heemskerk, where I grew up.

People from Heemskerk are traditionally known as donkeys. One theory is that this is because of their stubbornness.

My stubbornness brought me into academia, and so far it has kept me there.



Donkey statue in Heemskerk

Starting in computer science

I did my bachelor's degree in Computer Science at the University of Amsterdam.

I was most interested in abstract topics, such as the theory of algorithms and the formal semantics of programming languages.

In my bachelor's thesis, I studied an algebraic semantics for a simple formal programming language.

My supervisors, ALBAN PONSE and INGE BETHKE, encouraged me to apply to the Master of Logic programme at the Institute for Logic, Language and Computation in Amsterdam.

Continuation in logic

I was admitted to the Master of Logic and greatly enjoyed the programme.

I was surrounded by motivated people from many different backgrounds, and I encountered mathematical, computational, linguistic, and philosophical approaches to logic.

Under the supervision of NICK BEZHANISHVILI, I wrote my master's thesis on canonical formulas for an intuitionistic modal logic called lax logic.

The method of canonical formulas

Recall that intermediate logics lie between intuitionistic propositional logic **IPC** and classical propositional logic **CPC**.

Their uniform axiomatization was a major problem until ZAKHARYASCHEV introduced the method of canonical formulas in the late 1980s and early 1990s.

The method relies on two steps:

1. describe the failure of a formula by finitely many *geometric refutation patterns*;
2. encode these patterns by *canonical formulas*.

Using this method, ZAKHARYASCHEV showed that every intermediate logic can be axiomatized uniformly by formulas describing such finite patterns.

Canonical formulas and modal companions

He later extended the method to modal logics above **K4**, that is, to transitive modal logics.

This extension is related the close connection between intermediate logics and their modal companions.

Canonical formulas have been used to describe this connection. For example:

- They gave a positive answer to the DUMMETT–LEMMON conjecture: an intermediate logic is Kripke complete if and only if its least modal companion is.
- They provided a new proof of the BLOK–ESAKIA theorem: the lattice of intermediate logics is isomorphic to the lattice of modal logics extending GRZEGORCZYK logic.

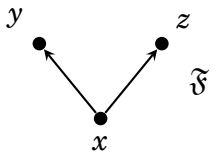
An example

A simple example comes from the GÖDEL–DUMMETT logic:

$$\mathbf{LC} = \mathbf{IPC} \oplus \big((\varphi \to \psi) \vee (\psi \to \varphi)\big).$$

In its Kripke frames, two points above the same point must be comparable.

The simplest obstruction is therefore the three-point fork $\mathfrak{F}$:



The canonical formula $\alpha(\mathfrak{F})$ expresses that this pattern is forbidden. Hence, $\mathbf{LC} = \mathbf{IPC} \oplus \alpha(\mathfrak{F})$.

Propositional lax logic

Propositional lax logic adds a modal operator $\bigcirc$ to intuitionistic propositional logic.

$$\varphi \to \bigcirc\varphi, \qquad \bigcirc\bigcirc\varphi \to \bigcirc\varphi, \qquad \bigcirc\varphi \wedge \bigcirc\psi \to \bigcirc(\varphi \wedge \psi).$$

The same operator has appeared in several different settings:

- $\bigcirc\varphi$ means that *φ is locally true*; (Goldblatt, 1981)
- $\bigcirc\varphi$ means that *φ holds under some constraint*; (Fairtlough and Mendler, 1997)
- $\bigcirc\varphi$ represents a *computation producing a value of type φ*. (Benton, Bierman, and de Paiva, 1998)

The obstacle for lax logic

The method of canonical formulas needs a finite description of every refutation.

Algebraically, this is often obtained from a locally finite reduct (finitely many generators produce only finitely many elements).

For intuitionistic modal logics, such a reduct is usually not available.

However, for lax logic forgetting implication gives a locally finite reduct.

Canonical formulas for lax logic

This observation allowed me to extend the method of canonical formulas to lax logic.

Theorem (SDM, 2020)

Every extension of propositional lax logic is axiomatizable by canonical formulas.

Application (SDM, 2020)

If an intermediate logic is

- tabular,
- has the finite model property, or
- Kripke complete, ("Lax" DUMMETT–LEMMON)

then its least lax extension has the same property.

From logic to duality

A key tool in this work was ESAKIA duality. It represents Heyting algebras by ordered topological spaces.

I was fascinated by how duality turns algebraic questions into geometric ones.

I therefore pursued a PhD in Mathematics with GURAM BEZHANISHVILI, a former student of LEO ESAKIA, at New Mexico State University.



The Organ Mountains above New Mexico State University

Pointfree topology and Priestley duality

During my PhD, I studied pointfree topology using Priestley duality.

Pointfree topology is an algebraic approach to topology.

Recall that a topological space (X, τ) consists of a set X together with a topology τ. Informally, pointfree topology forgets X and keeps only τ, viewed as a certain lattice. The resulting object is called a locale.

This approach can avoid some set-theoretic problems and some uses of the Axiom of Choice.

For example, there is no pointfree BANACH–TARSKI paradox, and the pointfree version of TYCHONOFF's theorem can be proved without the Axiom of Choice.

Locales without enough points

The disadvantage is that some of the usual geometric intuition can disappear.

Every topological space gives a locale of open sets, and every locale has an associated space of points.

However, this correspondence is not perfect: the points of a locale do not always determine the locale completely.

Some locales even have no points at all. They therefore cannot be represented by an ordinary topological space.

So how can we recover a geometric picture of an arbitrary locale?

Recovering the geometry

In my PhD, I used Priestley duality to put points back into pointfree topology.

Priestley duality represents every bounded distributive lattice by an ordered topological space. Since locales are special bounded distributive lattices, Priestley duality also applies to them.

This gives a geometric picture even when the locale itself has too few points, or no points at all.

I then studied how properties of the locale are reflected in the topology and order of this space.

This work led to several journal papers and continuing international collaborations.

A postdoc in Italy

After my PhD, I became a postdoctoral researcher in the Department of Mathematics at the University of Salerno, Italy.



Somewhere near Fisciano, Salerno, Italy

MV-algebras

At the University of Salerno I worked with LUCA SPADA on the spectrum problem for MV-algebras.

MV-algebras provide the algebraic semantics of LUKASIEWICZ many-valued logic.

Being part of Salerno's algebraic logic group encouraged me to return more directly to logic.

In particular, I became interested again in the programme on canonical formulas that I had begun during my master's degree.

Proposed research

My proposed research returns to **canonical formulas for intuitionistic modal logics**.

I plan to develop the construction for **IK4** that I outlined in a talk in 2021, and to explore how far the method can be extended.

This connects naturally with MINGHUI MA's work on related intuitionistic modal logics.

More broadly, I hope to combine my earlier work in logic with the algebraic and duality-theoretic methods developed during my PhD.

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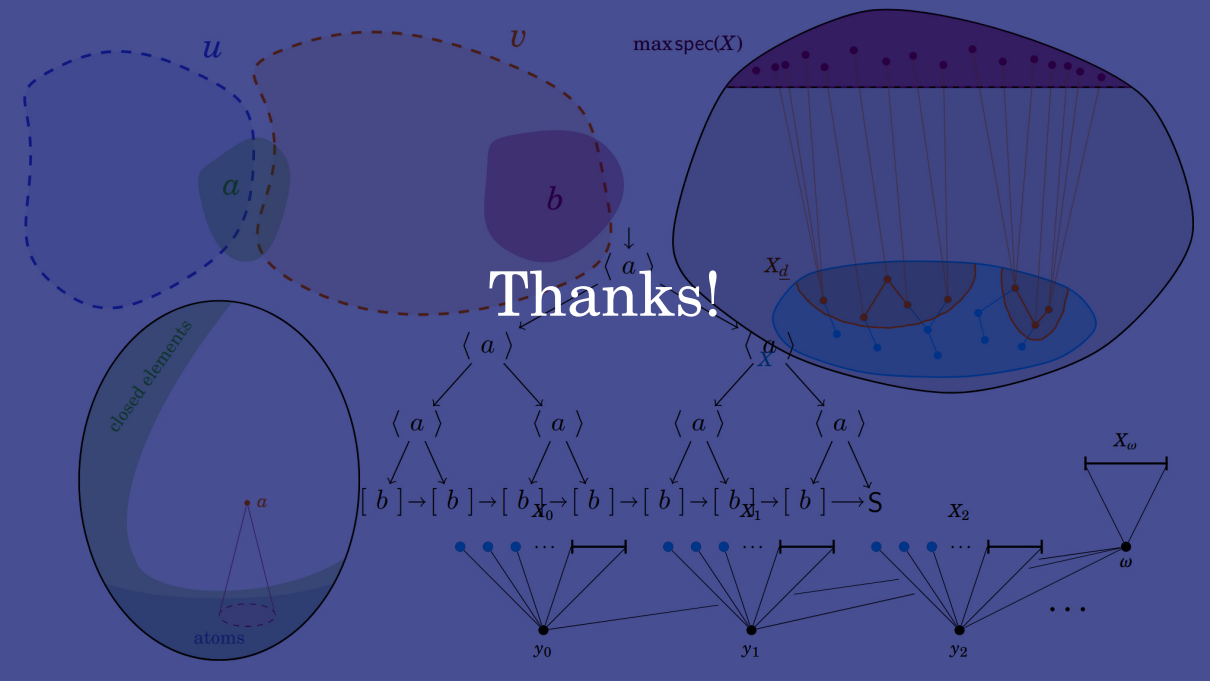
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Thanks!